Evaluation of the Characteristics of a Viscoelastic Material from Creep Analysis Using the Universal Viscoelastic Model. I. Isolation of the Elastic Component Designated as the "Projected Elastic Limit"

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Received 23 July 2002; accepted 12 November 2002

ABSTRACT: In a preceding publication this author introduced a new universal viscoelastic model to describe a definitive relationship between constant strain rate, creep, and stress relaxation analysis for viscoelastic polymeric compounds. One extremely important characteristic of this new model is that it also characterizes secondary creep very well. Because secondary creep is the linear portion of creep after the completion of primary creep, then a straight line with a slope and an intercept can describe secondary creep. To effectively define a straight line in the secondary creep region it was found necessary to obtain averages of the instantaneous slope and the instantaneous intercept strain by averaging over a series of equally spaced data points in the secondary slope region. Most importantly, this average intercept strain was found to be independent of creep stress and creep time. This means that all the secondary creep straight lines must pass through the same intercept creep strain for all creep stresses. The results presented in this study strongly indicate that this secondary creep intercept strain is independent of creep stress and creep time, and appears to increase as the value of the efficiency of yield energy dissipation decreases. Because a decrease in the efficiency of yield energy dissipation, n, appears to correlate with an increase in the elastic solid like character of a material, then it appears that this secondary creep intercept strain should be a direct measure of the strain that the material can survive to retain its full elastic character. Therefore, this secondary creep intercept strain has been designated as the "Projected Elastic Limit" of a given viscoelastic material. J Appl Polym Sci 89: 2923–2936, 2003

Key words: stress; srain; creep; relaxation; viscoelastic properties

INTRODUCTION

In recent years the need for a simple analysis approach that relates creep, stress relaxation, and constant strain rate measurements all in one simple model has been generated as a result of the extended use of finite element analysis involving polymeric compounds[1] and composites.[2] One such unifying model has recently been published by this author,[3] which introduced a new universal viscoelastic model to describe a definitive relationship between constant strain rate, creep, and stress relaxation analysis for viscoelastic polymeric compounds. Recently, a better understanding of the viscoelastic characteristics of this model relative to constant strain rate yield properties and stress relaxation has been further elucidated.[4] These new concepts of viscoelasticity introduced for this model were consistent with the earlier efforts of Scott Blair[5] as well as the more recent efforts of Hernandez et al.[6] In addition, this model has also recently been extended[7] to better understand the similarities of the failure criterion characteristics involving the strain at critical creep and the strain at yield for constant strain rate measurements.

Prior to the introduction of this new universal viscoelastic model several authors had attempted to describe two or more of these viscoelastic concepts in one unifying formulation.[8,9] However, most of the effort over the years has been to simulate uniaxial creep,[10,11] stress relaxation,[8] or constant strain rate data[12–15] separately. This new formulation approach offers a reasonably simple process in which to shift from a constant strain rate configuration to a creep calculation or stress relaxation configuration without changing formulation considerations or without stress or strain discontinuities.

All three phases of the creep curve including primary, secondary, and tertiary creep have been well represented using this new model. One extremely important attribute of this new model is that it also characterizes secondary creep very well. As a result of this, some surprisingly important new viscoelastic material characteristics of a material have been eluci-

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dated directly from a careful analysis of the creep properties of a material.

This study will show how these new viscoelastic characteristics can be generated from creep measurement evaluations using this new model. For reference, this new universal viscoelastic model[3] will be briefly reviewed before introducing a new approach to evaluate the "Projected Elastic Limit" from creep measurements.

BRIEF REVIEW OF THE UNIVERSAL VISCOELASTIC MODEL RELATING CONSTANT STRAIN RATE, CREEP, AND STRESS RELAXATION MEASUREMENTS

The basic new viscoelastic model recently published elsewhere[3] begins with the most general equation to fit a stress–strain curve, which can be written as

$$\frac{\sigma}{\sigma_y} = K\varepsilon + A_2(K\varepsilon)^2 + A_3(K\varepsilon)^3 + A_4(K\varepsilon)^4 \ldots + A_n(K\varepsilon)^n \quad (1)$$

where $K = E/\sigma_y \sim$ constant for a series of strain rates for the same polymer formulation, and $A_2, A_3, \ldots A_i$ = variable constants for a series of strain rates for the same polymer formulation.

In this study only the first three constants in eq. (1) have been addressed as:

$$\frac{\sigma}{\sigma_y} = K\varepsilon + A_2(K\varepsilon)^2 + A_3(K\varepsilon)^3 \quad (2)$$

According to Brown[13,17,18] and several other authors,[14,19] $K = E/\sigma_y$ is normally a constant for a given polymer formulation that typically ranges from 40–60. The two other conditions required to evaluate the constants A_2 and A_3 in eq. (2) would include the following:

By definition,

$$\sigma = \sigma_y, \text{ when } \varepsilon = \varepsilon_y$$

Second condition,

$$d\sigma/d\varepsilon = 0 \text{ at } \sigma = \sigma_y \text{ when } \varepsilon = \varepsilon_y \quad (3)$$

Using these conditions it can be shown that if $K\varepsilon_y \leq 3$

$$A_2 = \frac{(3 - 2K\varepsilon_y)}{K^2\varepsilon_y^2} \quad (4)$$

$$A_3 = \frac{(K\varepsilon_y - 2)}{K^3\varepsilon_y^3} \quad (5)$$

Thus, if $[d(\sigma/\sigma_y)/d\varepsilon 0 = 0$, and if $K\varepsilon_y \leq 3$, then the two extrema at $\varepsilon = \varepsilon_1$ and $\varepsilon = \varepsilon_2$ can be found to yield a maximum at

$$\sigma_1 = \sigma_y \quad \text{at} \quad \varepsilon_1 = \varepsilon_y \quad (6)$$

and a minimum at

$$\sigma_2 = \sigma_y\left(\frac{K^2\varepsilon_y^2(4K\varepsilon_y - 9)}{27(K\varepsilon_y - 2)^2}\right) \text{ at } \varepsilon_2 = \varepsilon_y\left(\frac{K\varepsilon_y}{(3K\varepsilon_y - 6)}\right) \quad (7)$$

The relationship between yield stress, σ_y, and time to yield, t_y, can be addressed using the following simple relationship currently included in ASTM D2837-98a (Standard Test Method for Obtaining Hydrostatic Design Basis for Thermoplastic Pipe Materials):

$$\sigma_y = \frac{\beta}{t_y^n} \quad (8)$$

where σ_y is the engineering yield stress, t_y is the time to yield, n is the efficiency of yield energy dissipation, and β is the constant. This relationship has also been used by Reinhart[16] to predict long-term failure stress (which is normally close to the stress evaluated from the stress relaxation of the yield stress) as a function of time.

The calculated values of strain, ε, can also be evaluated on a time scale by noting that the time, t, to reach a given strain, ε, can be evaluated from the characteristic strain rate, $\dot{\varepsilon}_i$, , as:

$$t = \frac{\varepsilon}{\dot{\varepsilon}_i} \quad (9)$$

Also note that the yield strain, ε_y, and the time to yield, t_y, are also related by a characteristic strain rate, $\dot{\varepsilon}_i$,, as:

$$t_y = \frac{\varepsilon_y}{\dot{\varepsilon}_i} \quad (10)$$

Preliminary experimental measurements by this author as well as others in the literature[8,9] have found that the strain at yield, ε_y, generally has been found to be a linear function of the characteristic strain rate, $\dot{\varepsilon}_i$, for constant strain rate measurements as:

$$\varepsilon_y = \varepsilon_\infty + \alpha\dot{\varepsilon}_i \quad (11)$$

where ε_∞ is the limiting strain to yield when the strain rate approaches an infinitely small value $((\dot{\varepsilon}_i \to 0)_i \to 0)$, and α is a constant.

Brinson and DasGupta[8] point out that Crochet[20] predicted theoretically that the yield strain should decrease with an increase in strain rate. As indicated previously,[3] this author has found that α is indeed negative for polyethylene. However, Malpass[9] and this author have found that for most ABS materials the strain to yield often increases as the strain rate increases, which would make α positive. In addition, Brinson and DasGupta[8] also found out experimentally that the yield strain increased with an increase in strain rate for polycarbonate.

However, it should be pointed out that the linear function described by eq. (11) appears to be a good approximation of the more detailed model at low strain rates. For the full range of strain rates and particularly for large strain rates, then eq. (11) is better described by the following equation:

$$\varepsilon_y = \varepsilon_\infty + \varepsilon_0(1 - \mathbf{e}^{-\gamma\dot{\varepsilon}_i}) \tag{12}$$

This equation has the following limits

$$\begin{aligned} \varepsilon_y &\rightarrow \varepsilon_\infty \quad \text{as} \quad \dot{\varepsilon}_i \rightarrow 0 \text{ (very long times)} \\ \varepsilon_y &\rightarrow \varepsilon_\infty + \varepsilon_o \quad \text{as} \quad \dot{\varepsilon}_i \rightarrow \infty \text{ (very short times)} \end{aligned}$$

In addition, eq. (12) can also be simplified using a MacLaurin series of the exponential term to give

$$\varepsilon_y = \varepsilon_\infty + \varepsilon_o\left(\gamma\dot{\varepsilon}_i - \frac{\gamma^2\dot{\varepsilon}_i^2}{2} + \cdots\right) \tag{13}$$

When $\dot{\varepsilon}_i \ll 1_i << 1$, then eq. (13) reduces to

$$\varepsilon_y = \varepsilon_\infty + \varepsilon_o\gamma\dot{\varepsilon}_i \tag{14}$$

Notice that eq. (14) is exactly the same as eq. (11) if

$$\alpha = \varepsilon_o\gamma \tag{15}$$

In this study, an ABS polymer will be used as an example of a viscoelastic material to illustrate the capabilities of the models presented. Based on unpublished constant strain rate measurements made by this author, the ABS material to be used as an example in this study will utilize the following constants ($\gamma = 50$ min, $\varepsilon_o = 0.0044$ and $\varepsilon_\infty = 0.04$).

Substituting eq. (10) into eq. (8) then gives

$$\sigma_y = \beta\left(\frac{\dot{\varepsilon}_i}{\varepsilon_y}\right)^n \tag{16}$$

Equation (16) can then be substituted into eq. (2) to give

$$\sigma = \beta\left(\frac{\dot{\varepsilon}_i}{\varepsilon_y}\right)^n[K\varepsilon + A_2(K\varepsilon)^2 + A_3(K\varepsilon)^3] \tag{17}$$

Although the yield strain, ε_y, is best described over the full range of strain rates by eq. (12), it is often convenient to use eq. (11) to simulate the yield strain, ε_y, at very low strain rates to give a simplified form of both eqs. (16) and (17) as

$$\sigma_y = \beta\left(\frac{\dot{\varepsilon}_i}{\varepsilon_\infty + \alpha\dot{\varepsilon}_i}\right)^n \tag{18}$$

$$\sigma = \beta\left(\frac{\dot{\varepsilon}_i}{\varepsilon_\infty + \alpha\dot{\varepsilon}_i}\right)^n[K\varepsilon + A_2(K\varepsilon)^2 + A_3(K\varepsilon)^3] \tag{19}$$

Based on eqs. (16)–(19), it is apparent that any tensile stress, σ, associated with a specific strain value, ε, including the yield strength, σ_y, will increase with an increase in the strain rate, $\dot{\varepsilon}_i$. However the strain to yield, ε_y, based on either eq. (11) or eq. (12) is only mildly sensitive to strain rate, and is allowed to either increase or decrease slightly with an increase in the strain rate, $\dot{\varepsilon}_i$.

It is also interesting to address the case that exists at long times, t, or using eq. (19) at very low elongation rates, $\dot{\varepsilon}_i$. For this case, note that the yield stress, ε_y, approaches a limiting value, ε_∞:

$$\varepsilon_y = \varepsilon_\infty + \alpha\dot{\varepsilon}_i \rightarrow \varepsilon_\infty \quad \text{as} \quad \dot{\varepsilon}_i \rightarrow 0$$

For this case the constants A_2 and A_3 also approach the following values

$$A_2' = \frac{(3 - 2K\varepsilon_\infty)}{K^2\varepsilon_\infty^2} \tag{20}$$

$$A_3' = \frac{(K\varepsilon_\infty - 2)}{K^3\varepsilon_\infty^3} \tag{21}$$

and eq. (19) reduces to

$$\sigma = \beta\left(\frac{\dot{\varepsilon}_i}{\varepsilon_\infty}\right)^n[K\varepsilon + A_2'(K\varepsilon)^2 + A_3'(K\varepsilon)^3] \tag{22}$$

Combining eqs. (9) and (22) gives

$$\sigma = \beta\left(\frac{\varepsilon}{\varepsilon_\infty}\right)^n\left(\frac{1}{t^n}\right)[K\varepsilon + A_2'(K\varepsilon)^2 + A_3'(K\varepsilon)^3] \tag{23}$$

Again, it should be noted that eqs. (22) and (23) apply only to the condition where the yield strain, ε_y, approaches its limiting value of ε_∞ as a result of the strain rate, $\dot{\varepsilon}_i$, approaching zero (0). Modification of eq. (23) can also be rearranged for creep analysis in the following form:

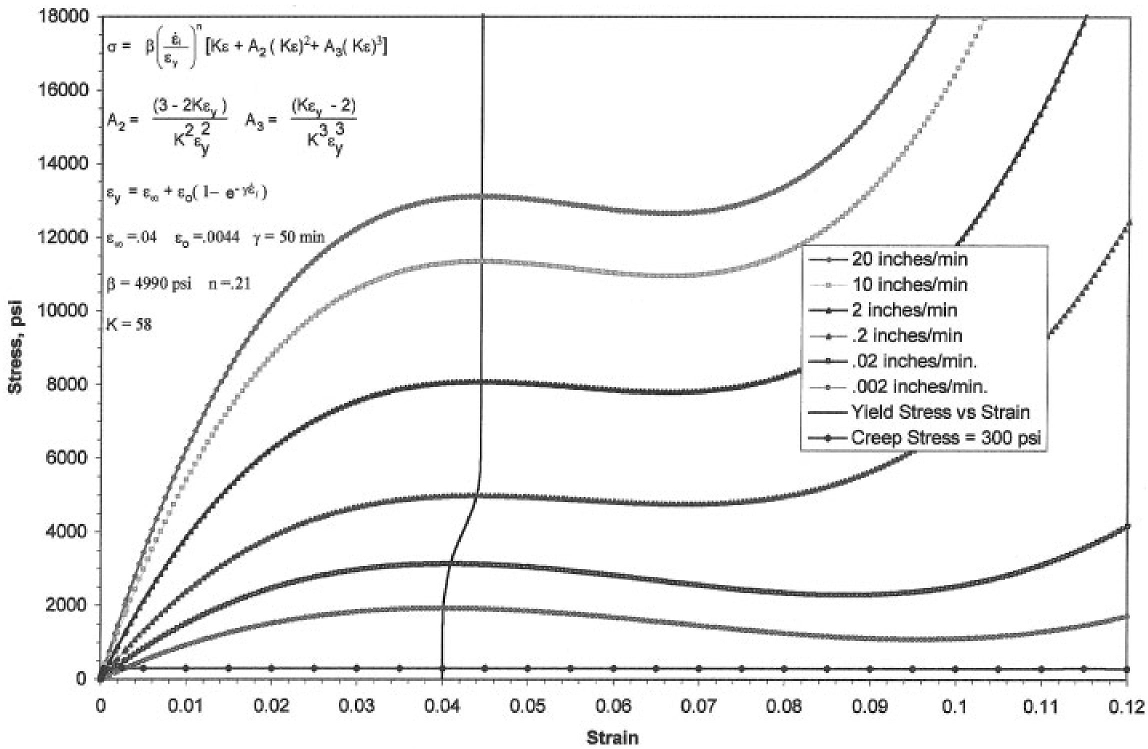

Figure 1 Calculated engingeering stress vs. strain at various strain rates for a simulated ABS material using the universal viscoelastic model with indications of creep at 300 psi.

$$t = \left(\frac{\varepsilon}{\varepsilon_\infty}\right)\left(\frac{\beta}{\sigma}\right)^{1/n} [K\varepsilon + A_2'(K\varepsilon)^2 + A_3'(K\varepsilon)^3]^{1/n} \quad (24)$$

As was indicated in a previous publication,[3] eqs. (22), (23), and (24) can be extremely helpful when trying to address either creep or stress relaxation at very low strain rates, $\dot{\varepsilon}_i$, or at very long times, t.

In general, eqs. (1)–(19) can be used to describe a complete series of uniaxial constant strain rate curves for a given polymer formulation and/or processing condition, as indicated for an example ABS type material in Figure 1. These stress vs. strain curves have been calculated primarily using eq. (17) from this new universal viscoelastic model for a series of strain rates from 0.002 in/min to 20 in/min. The yield stress vs. yield strain curve also indicated in Figure 1 was calculated primarily from eqs. (17) and (12), respectfully. For reference, all of the constant strain rate curves in Figure 1 were generated using eq. (17) with the following typical parameters for an ABS type polymeric material $K = 58$, $\gamma = 50$ min, $\varepsilon_o = 0.0044$, and $\varepsilon_\infty = 0.04$, $\beta = 4990$ psi and $n = 0.21$. Again, these constants represent typical values for an ABS material as obtained from unpublished constant strain rate data generated by this author. As described in a previous publication,[3] a creep curve at a constant stress such as 300 psi as indicated in Figure 1 can be developed from an identification of the strain at a series of points at the same stress level but from a series of constant strain rate curves.

The stress vs. strain curves in Figure 1 have been converted to stress vs. time curves in Figure 2 using eq. (9). The values for the yield stress vs. time to yield in Figure 2 have been obtained by applying eq. (8) or by applying eq. (10) to the strain-to-yield values shown in Figure 1. For reference, the yield stress vs. time to yield curve in Figure 2 was again generated using the following typical parameters for an ABS type polymeric material with $\beta = 4990$ and $n = 0.21$. Again, notice the locus of points that would make up the creep curve at 300 psi as indicated in Figure 2. The generation of a creep curve from these data points will be discussed in more detail in the next section.

CREEP CURVE GENERATION USING THE NEW VISCOELASTIC MODEL

Creep is defined as the time-dependent increase in strain of a viscous or viscoelastic material under sustained and constant stress. As indicated in Figure 1, a creep curve can be developed from an identification of the strain at a series of points at the same stress level from a series of constant strain rate curves. However, initially the increase in stress to the level from which the creep curve can be initiated must be simulated.

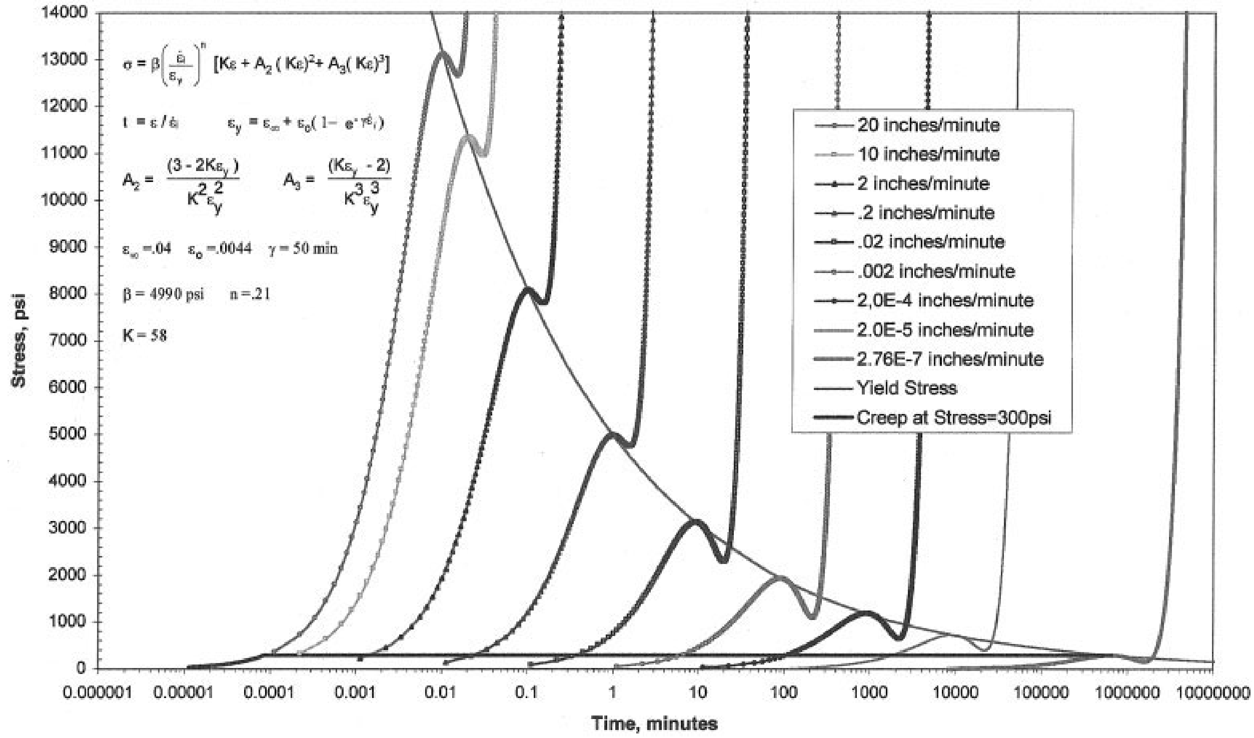

Figure 2 Calculated stress vs. time at different strain rates using the universal viscoelastic model with indications of creep at 300 psi and yield stress relaxation.

Typically, the simplest simulation approach can be achieved from a constant strain rate process that can be used to arrive at the desired level of unchanging stress, σ_C, from which the creep process can begin. Equation (17) can then be used to generate this stress, σ, vs. strain, ε, curve at a specific strain rate, $\dot{\varepsilon}_i$, until the desired level of stress, σ_C, is achieved from which a creep process can be initiated.

Once the desired stress level, σ_C, has been reached using a constant strain rate approach, then eq. (19) must be solved for the strain to give a specific stress, σ_C, as the strain rate, $\dot{\varepsilon}_i$, is continued to be decreased to get to longer creep strains, ε_C, which can then be converted to creep times, t_C. Also note that after a designated very low effective strain rate, $\dot{\varepsilon}_i$, is reached, the creep time, t_C, accumulated for a creep strain, ε_C, can be calculated directly from eq. (24). The locus of points involving calculated values of creep strain, ε_C, and the associated creep times, t_C, then constitutes the creep curve as indicated in Figure 3.

There are four potential ways then to calculate creep strain, ε_C, using eqs. (17) or (19) as a function of strain rate, $\dot{\varepsilon}_i$, to longer creep times, t_C. These four options include:

1. Solve eqs. (17) or (19) as a cubic equation to calculate the appropriate creep strain, ε_C, at decreasing levels of strain rate, $\dot{\varepsilon}_i$, but at the desired creep stress level, σ_C. The creep time, t_C, accumulated for a specific creep strain, ε_C, at a specific strain rate, $\dot{\varepsilon}_i$, can then be calculated directly from eq. (9).
2. Solve eqs. (17) or (19) using a numerical method such as the Newton-Raphson method to calculate the appropriate creep strain, ε_C, at decreasing levels of strain rate, $\dot{\varepsilon}_i$, but at the desired creep stress level, σ_C. The creep time, t_C, accumulated for a specific creep strain, ε_C, at a specific strain rate, $\dot{\varepsilon}_i$, can then be calculated directly from eq. (9).
3. Solve eqs. (17) or (19) as a constant strain rate evaluation for each strain rate, $\dot{\varepsilon}_i$, and then solving the eqs. (17) or (19) by trial and error for the creep strain, ε_C, that yields the desired creep stress, σ_C. The creep time, t_C, accumulated for a specific creep strain, ε_C, at each specific strain rate, $\dot{\varepsilon}_i$, can then be calculated directly from eq. (9).
4. Assuming the controlling strain rates, $\dot{\varepsilon}_i$, are very small after the constant level of creep stress, σ_C, is achieved, and assuming the relative insensitivity of the values of A'_2 and A'_3 to strain rate at that point, then the creep curve can be calculated by close approximation directly using eq. (24). By setting the creep stress, σ_C, to a constant value then the creep time, t_C, can be calculated as a function of creep strain, ε_C, using eq. (24).

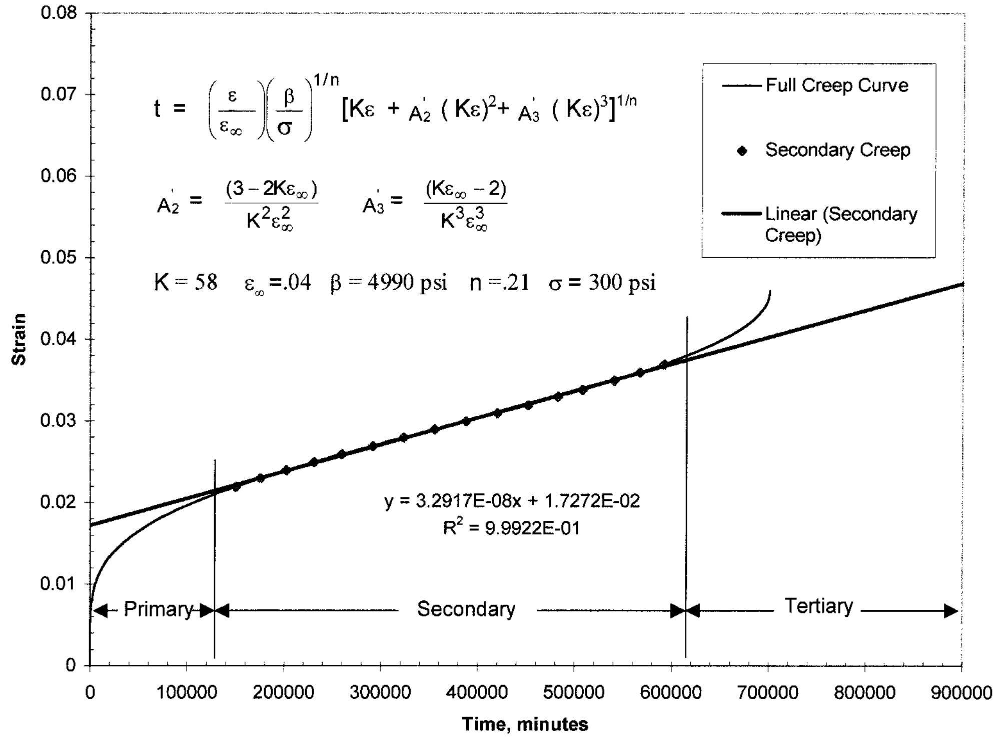

Figure 3 Calculated constant strain rate until the desired stress (300 psi) was reached followed by creep strain vs. time and showing all three phases of creep.

Although method 3 appears to be very time consuming, it can actually be evaluated relatively fast using a spreadsheet software such as MS Excel. This approach was also found to be particularly useful as the yield condition for creep or the inception of tertiary creep was approached and exceeded. If Option 3 is used, eqs. (17) or (19) is first applied at a constant strain rate to increase the stress and associated strain until the desired stress level has been achieved. After the desired stress level has been reached, the successive strain values for the creep process can be developed by identifying the appropriate strain on successive stress strain curves that corresponds to the desired level of stress being evaluated. At this point each constant strain rate curve can be used to generate only one strain level at a given stress level on the creep curve, as indicated in Figures 1 and 2. Because each constant strain rate curve can be described by eqs. (17) or (19), then this equation can be used to calculate the strain, ε_C, at the desired stress, σ_C, and at the characteristic strain rate of, $\dot{\varepsilon}_i$, being addressed. For a creep process, both the yield strain, ε_y, and the yield strength, σ_y, are functions of only the strain rate, $\dot{\varepsilon}_i$, as indicated in eqs. (12) and (16). Also the time, t_C, for a specific strain, ε_C, at a specific strain rate, $\dot{\varepsilon}_i$, can be calculated directly from eq. (9). Therefore, at a constant stress level, σ_C, the creep curve for a series of strain levels, ε_C, and their associated times, t_C, can be calculated from a series of constant strain rate stress–strain curves. The locus of these calculated points then constitutes all three phases of the creep curve including primary, secondary, and tertiary creep as indicated in Figure 3.

According to Thorkildsen,[17] Primary Creep includes all the initial changes in deformation prior to Secondary Creep. The first region of creep after Primary Creep that shows a linear increase in strain with time is called Secondary Creep. Tertiary Creep is the final stage of creep, and it will be shown that this stage of creep can be correlated with the yield point from constant strain rate measurements.

Using the formulation concepts to calculate creep as discussed in this article, the initial phase of a creep test begins with the constant strain rate component followed by the more typical creep process as indicated in Figure 3. Of particular interest is the observation that the three different phases of the creep curve in Figure 3 plot as straight lines when plotted on a log–log scale as indicated in Figure 4.

If the simplifying assumptions of Option 4 are acceptable after some limiting strain rate, then use of eq. (24) allows the simplest approach to generate all three phases of the creep curve including primary, secondary, and tertiary creep as indicated in Figure 4. In addition, the creep results in Figure 4 can be described over a much larger time scale in a very convenient fashion using Option 4. It is interesting to note in Figure 4 that both Option 3 and Option 4 give the

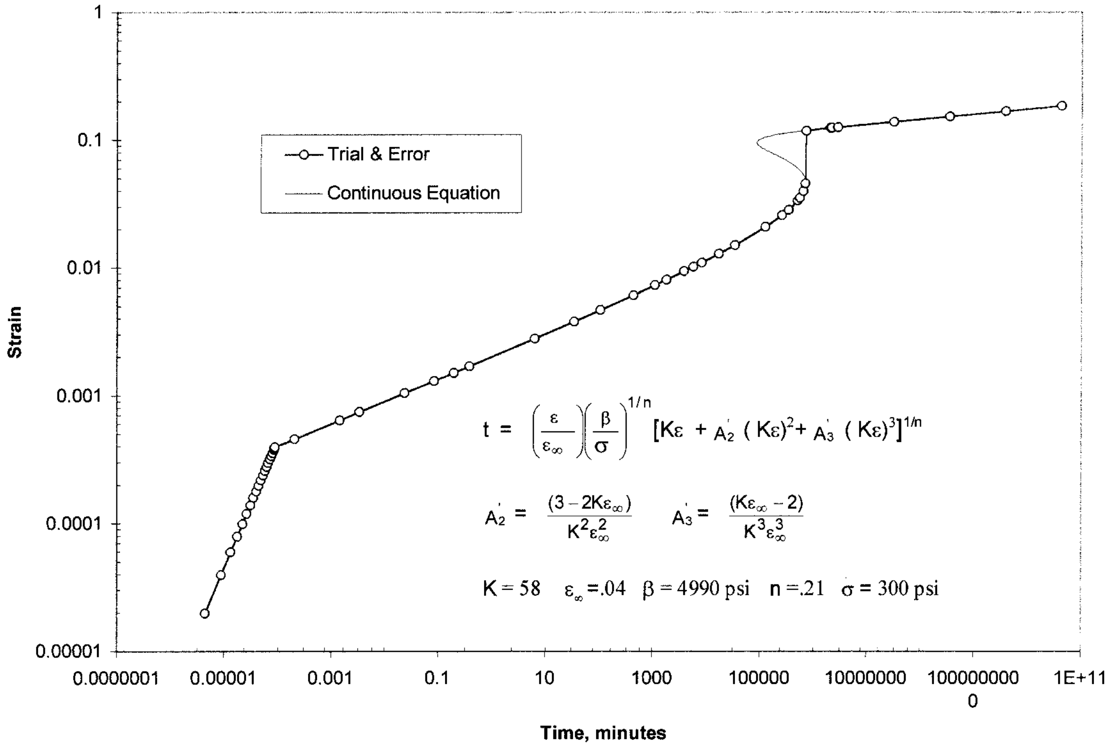

Figure 4 Calculated plot of long-term creep at 300 psi stress showing the linear character of strain at both short and long times on a log–log scale.

same creep curve up until the inception of tertiary creep. At this point there is a jump in the data using Option 3, but the results for Option 4 yield quite a different but continuous curve near the condition as defined by yield point failure criterion. An elucidation of the apparent conflict between Options 3 and 4 near creep failure as indicated in Figure 4 is easily understood. Because it is not possible to go back in time, then Option 3 is obviously the only realistic option for a realistic creep curve.

MODELING SECONDARY CREEP USING THE NEW VISCOELASTIC MODEL

As indicated in Figure 3, the evaluation of secondary creep would involve the region where the creep curve forms a straight line. By definition, a straight line for secondary creep would involve the following equation

$$\varepsilon = \left(\frac{d\varepsilon}{dt}\right)t + \varepsilon_I \tag{25}$$

The derivative defined by the slope indicated in eq. (25) would normally require a formulation where the strain, ε, would be a direct function of time, t. Even though we do not have a relationship with strain as a function of time, we do have eq. (24), which describes creep time, t, as a function of creep strain, ε. Therefore, the derivative of eq. (24) gives

$$\frac{dt}{d\varepsilon} = \frac{t}{\varepsilon}\left[1 + \frac{1}{n}\left(\frac{1 + 2A_2'(K\varepsilon) + 3A_3'(K\varepsilon)^2}{1 + A_2'(K\varepsilon) + A_3'(K\varepsilon)^2}\right)\right] \tag{26}$$

The reciprocal of eq. (26) then gives the desired derivative or slope as

$$\frac{d\varepsilon}{dt} = \frac{\varepsilon n}{t}\left(\frac{1 + A_2'(K\varepsilon) + A_3'(K\varepsilon)^2}{1 + n + (2 + n)A_2'(K\varepsilon) + (3 + n)A_3'(K\varepsilon)^2}\right) \tag{27}$$

It is apparent that the term $(d\varepsilon/dt)t$ can be conveniently obtained from eq. (27). A rearrangement of eq. (25) allows the direct calculation of the intercept strain value, ε_I, of the straight line as

$$\varepsilon_I = \varepsilon - \left(\frac{d\varepsilon}{dt}\right)t \tag{28}$$

Substituting eq. (27) into eq. (28) then gives

$$\varepsilon_I = \varepsilon\left(\frac{1 + 2A_2'(K\varepsilon) + 3A_3'(K\varepsilon)^2}{1 + n + (2 + n)A_2'(K\varepsilon) + (3 + n)A_3'(K\varepsilon)^2}\right) \tag{29}$$

Note that when $n = 0$ then eq. (29) reduces to

$$\varepsilon_I = \varepsilon \tag{30}$$

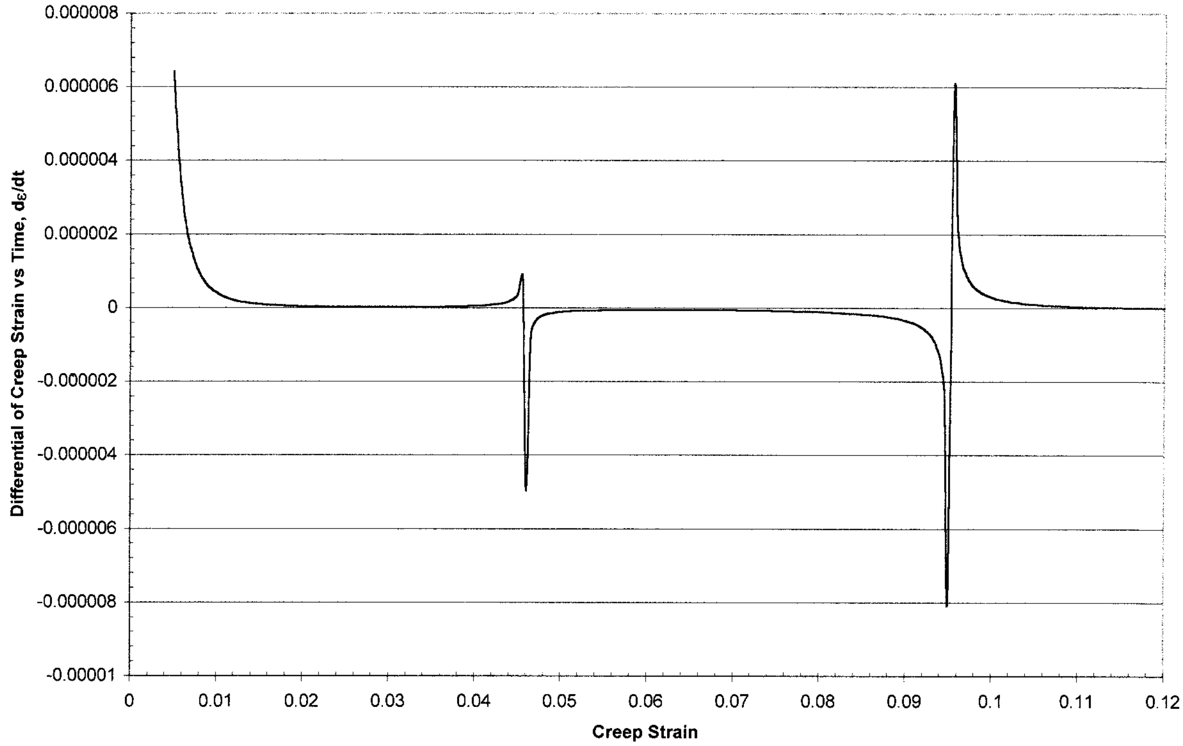

Figure 5 Differential (or slope) of creep strain vs. time, $d\varepsilon/dt$, vs. creep strain (creep stress = 300 psi).

This result indicates that when the efficiency of yield energy dissipation, n, is equal to zero ($n = 0$), then eq. (29) reduces to a condition where the instantaneous intercept strain is equal to strain. For such a material all strains would be within the elastic limit up to the failure condition. Such a material would probably then be best characterized as being a completely elastic material.

Equations (27) and (29) then represent the instantaneous slope and the instantaneous intercept at each creep strain, ε. It is particularly important to note that the instantaneous slope, as described by eqs. (24) and (27), is a function of the creep stress, σ, and creep strain, ε, plus all the variables indicated in Figure 1 including $K = 58$, $\varepsilon_\infty = 0.04$, $\beta = 4990$ psi, and $n = 0.21$. However, the instantaneous intercept defined by eq. (29) is only a function of creep strain, ε, and the constants $K = 58$, $\varepsilon_\infty = 0.04$, and $n = 0.21$. Most importantly, the intercept strain, ε_I, is independent of creep stress and creep time. This means that all the secondary creep straight lines must pass through the same intercept creep strain for all creep stresses. It is also clear from eq. (29) that the location of this intercept strain is also strongly dependent on the efficiency of yield energy dissipation, n, which has previously been shown[4] to be primarily a measure of the viscoelastic character of a material. We will expand further this important observation in the next sections of this article.

EVALUATION OF SECONDARY CREEP USING THE NEW VISCOELASTIC MODEL

As indicated in Figure 5, the instantaneous slope, $d\varepsilon/dt$, for creep at a constant stress of 300 psi has been generated by combining eqs. (25) and (27) for the same ABS example material. This instantaneous slope in Figure 5 is directly related to the slope of graphs in Figures 3 and 4. Note in Figure 5 that the derivative, $d\varepsilon/dt$, or creep slope approaches a nonzero minimum but nearly constant value in the secondary creep region, which appears to run from a strain of approximately $\varepsilon = 0.016$ to $\varepsilon = 0.04$.

The creep slope in Figure 5 also goes through zero and goes back to a slope of the opposite sign at a strains of approximately $\varepsilon_{CC} = 0.046$ and $\varepsilon = 0.95$. The exact location of these creep slope change locations can be obtained by simply setting the derivative described by eq. (26) to zero by setting $dt/d\varepsilon = 0$ and solving for the resulting equation for the strain at critical creep, ε_{CC} , to give

$$\varepsilon_{CC} = \left(\frac{-(n+2)A_2' \pm \sqrt{(n+2)^2 A_2'^2 - 4(n+1)(n+3)A_3'}}{2(n+3)A_3'K} \right) \tag{31}$$

Critical creep has been shown in a previous article[7] to be theoretically the maximum potential strain in creep and the strain at which failure in creep would be expected.

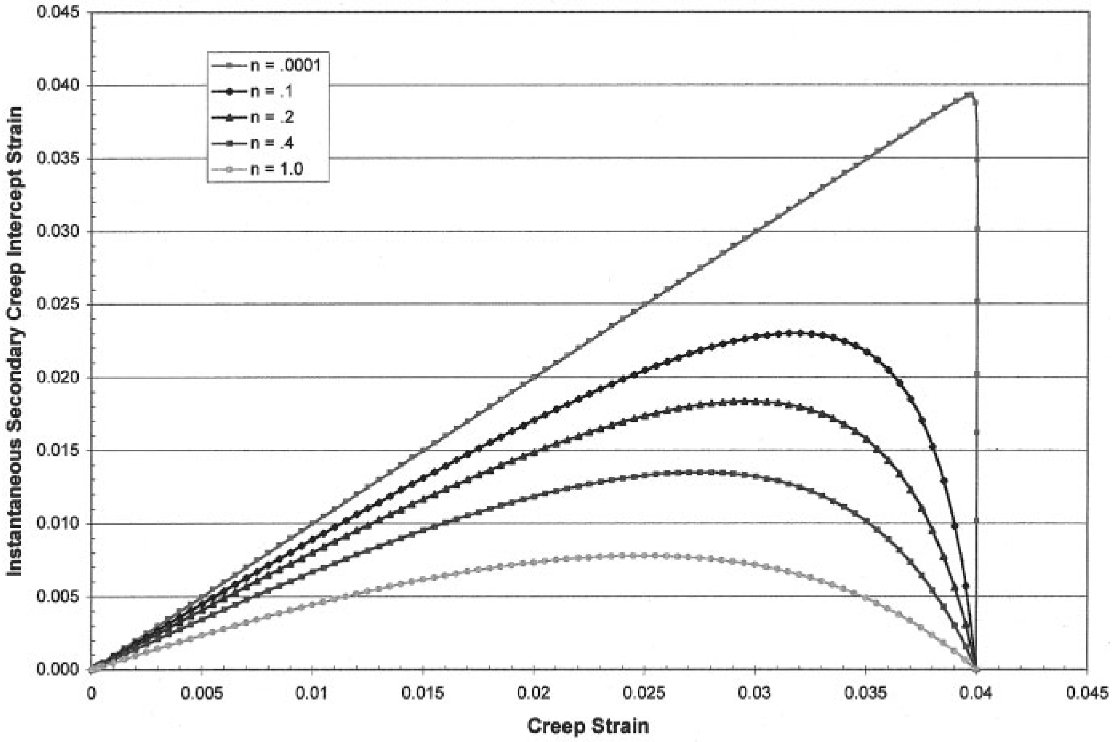

Figure 6 Instantaneous secondary creep intercept strain vs. creep strain using an example ABS material for the universal viscoelastic model at various levels of the efficiency of yield energy dissipation.

The average slope as indicated in Figure 3 must be obtained by averaging over a series of equally spaced data points in the secondary slope region such that

$$\left(\frac{d\varepsilon}{dt}\right)_{\text{Ave}} = \frac{\sum_{i=1}^{i=k}\left(\frac{d\varepsilon}{dt}\right)_i}{k} \tag{32}$$

Similarly, the instantaneous intercept strain, ε_I, shown in Figure 6 has been generated using eq. (29) at five different levels of the efficiency of yield energy dissipation from $n = 0.0001$ to $n = 1.0$. The creep strain at the maximum instantaneous intercept strain identifies the center of the secondary slope region of the creep curve. The creep strain at the maximum instantaneous intercept in Figure 6 also corresponds with a minimum in the instantaneous slope in the secondary slope region as indicated in Figure 5. Also note in Figure 6 that the magnitude of the maximum instantaneous intercept strain, ε_I, and the location of the center of the secondary slope region both shift with a change in the value of the efficiency of energy dissipation.

The shifts in the maximum instantaneous intercept strain and the center of the secondary slope region indicated in Figure 6 are more clearly visualized in Figure 7, where they are plotted directly as a function of the efficiency of yield energy dissipation, n. As indicated in Figure 7, both the magnitude of the maximum instantaneous intercept and the strain location at the center of the secondary slope region shift to lower values as the efficiency of energy dissipation increases. This result is consistent with Scott-Blair,[5] who argued that for a viscoelastic material a value of $n = 0$ should be more characteristic of an elastic solid and the a value of $n = 1$ should be more characteristic of a viscous liquid.

The average instantaneous intercept as indicated in Figure 3 again must also be obtained by averaging over a series of equally spaced data points in the secondary slope region such that

$$\varepsilon_{\text{IAve}} = \frac{\sum_{i=1}^{i=k} \varepsilon_{Ii}}{k} \tag{33}$$

Examples of secondary creep at different stress levels calculated for the example ABS material are shown in Figures 8–11. Several full creep curves have been developed in Figure 8 showing primary, secondary, and tertiary creep using the model developed in this study. It is very clear in Figure 8 that the location of the large increase in strain at critical creep, ε_{CC}, would most likely result in failure if the material did not fail readily at the inception of tertiary creep, which would be developed at a creep strain identified as $\varepsilon = \varepsilon_\infty$.

Note in Figure 9 that, as expected, the secondary creep curves for all stress levels converge at the same

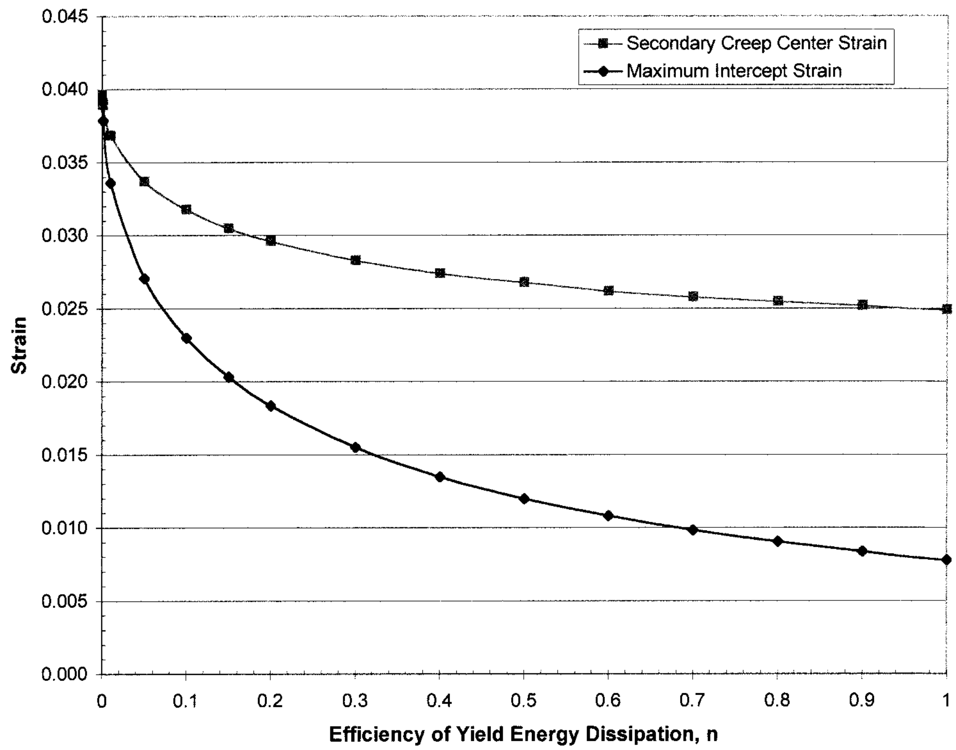

Figure 7 Maximum intercept strain and secondary creep center strain vs. efficiency of yield energy dissipation, n.

identical secondary creep intercept strain, ε_{IAve}. The linear secondary creep straight lines indicated in Figure 9 have been calculated using eqs. (27), (29), (31), and (32), and the results evaluated were the same as those included in Figure 8. In general, it has been found that the best linear secondary creep curve can

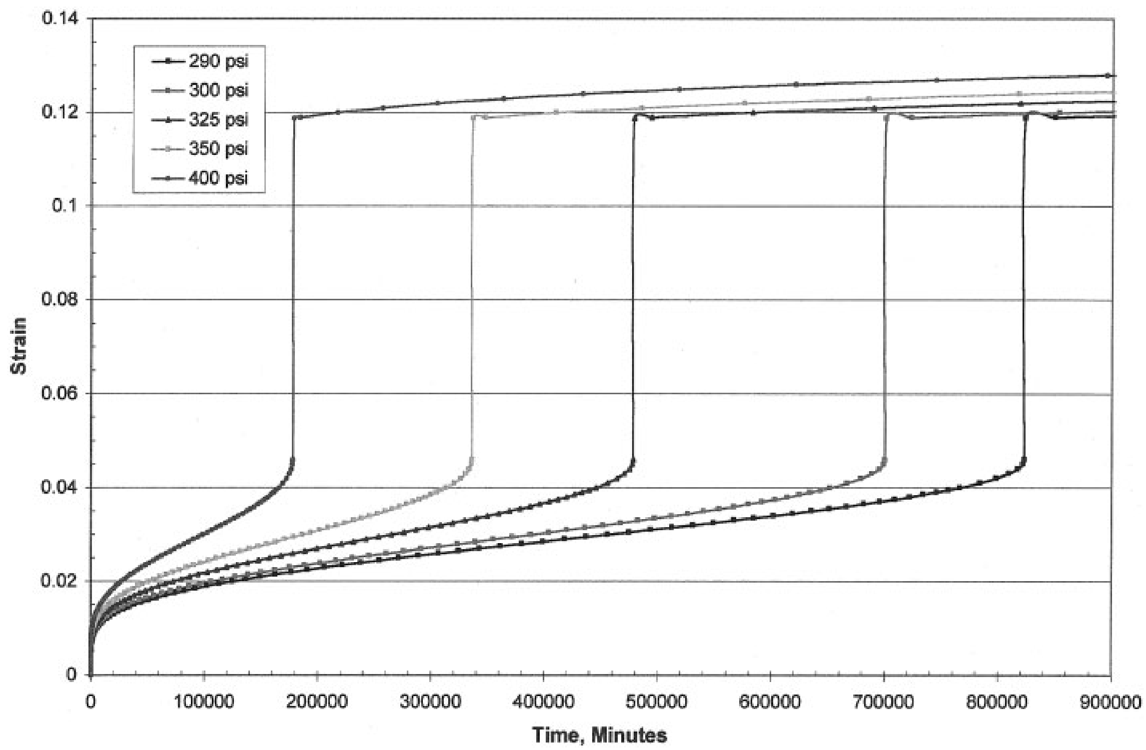

Figure 8 Creep strain vs. time for different levels of creep stress for the example ABS materials (n = 0.21).

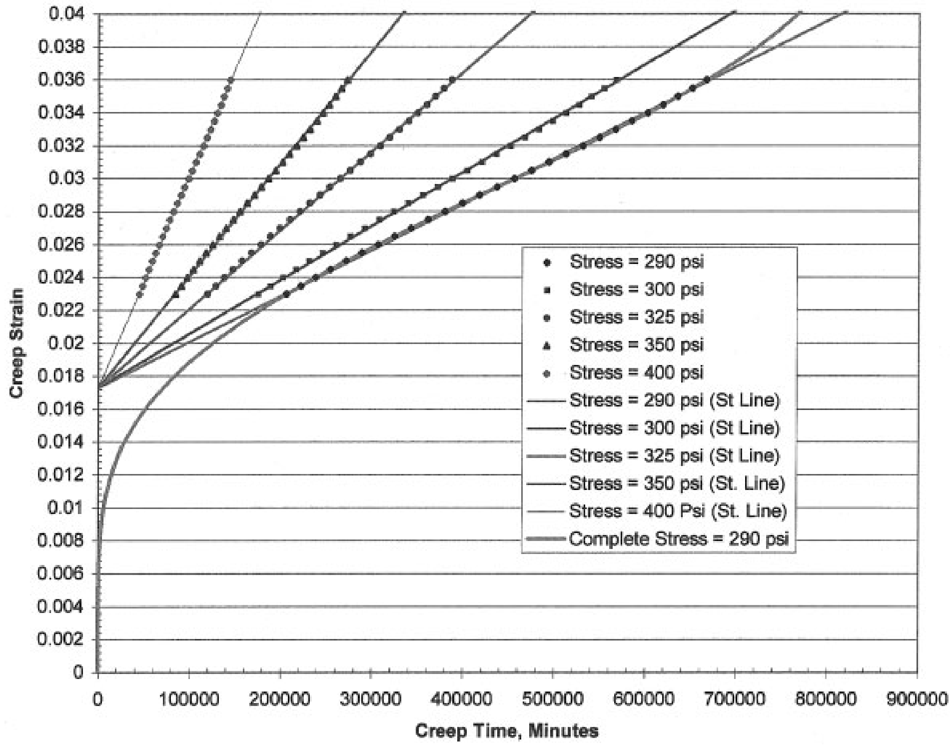

Figure 9 Secondary creep strain vs. time for different levels of creep stress for the example ABS material (n = 0.21).

be obtained by utilizing symmetric creep data evaluated within approximately 18% of the magnitude of the strain at the center of secondary creep.

The results in Figures 10 and 11 gave similar results to those already indicated for the secondary creep in Figure 9. However, the results in Figures 10 and 11 have been generated at two other efficiencies of yield energy dissipation (n = 0.15 and n = 1.0). The constant

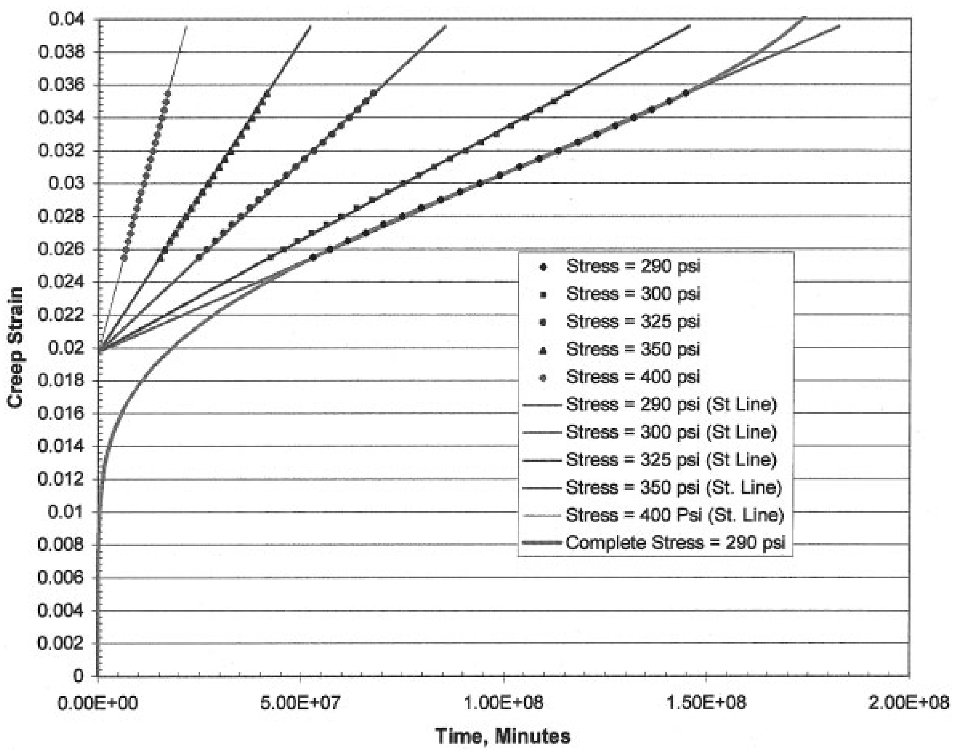

Figure 10 Secondary creep strain vs. time for different levels of creep stress for the example ABS material (n = 0.15).

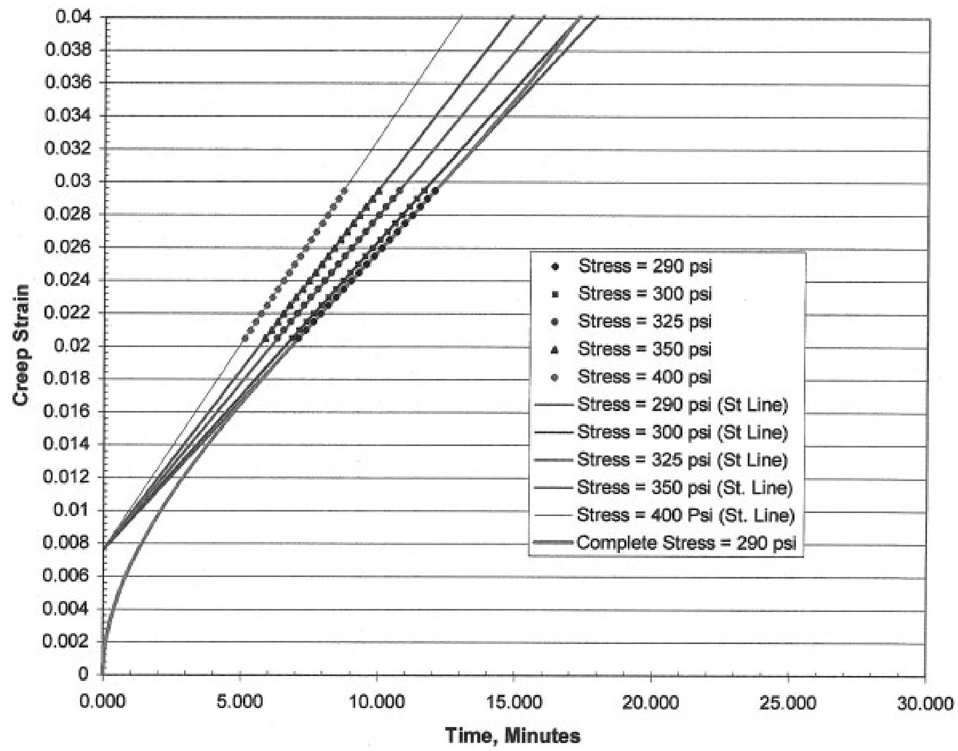

Figure 11 Secondary creep strain vs. time for different levels of creept stress for the example ABS material (n = 1.0).

projected strain intercepts from Figures 9–11 can be summarized as

Efficiency of yield energy dissipation, n	Projected strain intercept
$n = 0.15$	0.019776324
$n = 0.21$	0.017318915
$n = 1.0$	0.007649199

The results in Figures 6–11 appear to indicate the following:

1. If all other variables remain constant, then the linear secondary creep curves at all stress levels for a given viscoelastic material should terminate at exactly the same creep intercept strain.
2. The magnitude of this creep intercept strain appears to be inversely proportional to the efficiency of yield energy dissipation, n.
3. The value of the strain in the center of the secondary creep region and the maximum instantaneous creep intercept strain approach the limiting yield strain at very long times (ε_∞) as the efficiency of yield energy dissipation approaches zero.
4. In addition, as the efficiency of yield energy dissipation, n, approaches zero ($n = 0$), then the intercept appears to approach the strain and all strains for such a material appear to be within the elastic limit up to the failure condition.
5. As the value of the efficiency of yield energy dissipation, n, approaches 1.0 then the maximum creep intercept strain approaches zero.

An interpretation of these results will be addressed in the next section of this article.

UTILIZATION OF SECONDARY CREEP MEASUREMENTS TO IDENTIFY A MATERIALS "PROJECTED ELASTIC LIMIT"

In a recent review by this author[4] the power law constant, n, as used in the universal viscoelastic model, was found to be a dampening factor for the rate of dissipation of the available energy/volume relative to time in going from one strain rate curve to another at the yield condition. Consequently, the power law constant n was designated[4] the Efficiency of Yield Energy Dissipation with an effective range of $0 < n < 1$. Hernandez-Jimenez et al.[6] also recently reviewed several different models that have also tried to justify a theoretical development for the constant n. Most of these models referred back to the original model by Scott-Blair,[5] who justified the constant n from a fractional derivative for a viscoelastic material. If the material being described can be considered to be viscoelastic, then Scott-Blair argued that the value of n must exist in the range from 0(elastic solid) $< n < 1$(viscous liquid). Others have expanded on Scott-Blair's analysis as recently reviewed by Jimenez et al.[6]

The major advantage of the Scott-Blair analysis is that it does address a nice explanation of why the value of the "efficiency of yield energy dissipation," n, should only range from 0 to 1. It has been found previously[4,7] that if viscoelastic materials must survive considerable application stress for very long times they need to have "efficiency of yield energy dissipation" values of $n < 0.4$ to be practical. This result is certainly consistent with experimental data. An explanation for this phenomena based on the Scott-Blair analysis would describe such a viscoelastic material as having a more elastic solid like character than a viscous liquid-like character, because n would be closer to 0 than to 1.

The results presented in this study strongly indicate that the common secondary creep intercept strain, ε_{IAve}, indicated in Figures 9–11, appears to increase as the value of the efficiency of yield energy dissipation decreases. Because a decrease in the efficiency of yield energy dissipation, n, appears to correlate with an increase in the elastic solid like character of a material, then it appears that this secondary creep intercept strain, ε_{IAve}, should be a direct measure of the strain that the material can survive to retain its full elastic character. Therefore, until it can clearly be shown to be otherwise incorrect, this secondary creep intercept strain, ε_{IAve}, should probably be most correctly designated as the "Projected Elastic Limit" for a given viscoelastic material.

CONCLUSION

In a preceding publication this author introduced a new universal viscoelastic model to describe a definitive relationship between constant strain rate, creep, and stress relaxation analysis for viscoelastic polymeric compounds. All three phases of the creep curve including primary, secondary, and tertiary creep have been well represented using this new model. One extremely important characteristic of this new model is that it also characterizes secondary creep very well. Consequently, this study has introduced a new approach to evaluate the "Projected Elastic Limit" from secondary creep measurements.

Because secondary creep is the linear portion of creep after the completion of primary creep, then secondary creep can be described by a straight line with a slope and an intercept. It was shown in this study that both the instantaneous slope, $d\varepsilon/dt$, and the instantaneous intercept strain, ε_I, could easily be calculated utilizing the mathematical relationships developed from this new model. However, to effectively define a straight line in the secondary creep region it was found necessary to obtain averages of the instantaneous slope and intercept by averaging over a series of equally spaced data points in the secondary slope region. It is particularly important to note that the average slope in the secondary creep region was found to be a function of the creep stress, σ, and creep strain, ε, plus several other variables including K, ε_∞, β, and the efficiency of yield energy dissipation, n. However, the average intercept strain from secondary creep was found to be only a function of creep strain, ε, and the constants K, ε_∞, and n. Most importantly, this average intercept strain, ε_I, was found to be independent of creep stress and creep time. This means that all the secondary creep straight lines must pass through the same intercept creep strain for all creep stresses.

In a recent review by this author the power law constant, n, was found to be a dampening factor for the rate of dissipation of the available energy/volume relative to time in going from one strain rate curve to another at the yield condition. Consequently, the power law constant n was designated the Efficiency of Yield Energy Dissipation with an effective range of $0 \leq n \leq 1$. If the material being described can be considered to be viscoelastic, then Scott-Blair argued that the value of n must exist in the range from 0(elastic solid) $< n <$ 1(viscous liquid).

The results presented in this study strongly indicate that the secondary creep intercept strain, $_{IAve}$, is independent of creep stress and creep time and appears to increase as the value of the efficiency of yield energy dissipation decreases. Because a decrease in the efficiency of yield energy dissipation, n, appears to correlate with an increase in the elastic solid-like character of a material, it then appears that this secondary creep intercept strain, ε_{IAve}, should be a direct measure of the strain that the material can survive to retain its full elastic character. Therefore, until it can clearly be shown to be otherwise incorrect this secondary creep intercept strain, ε_{IAve}, should probably be most correctly designated as the "Projected Elastic Limit" of a given viscoelastic material.

If this "Projected Elastic Limit" can indeed be characterized by the strain ε_{IAve}, then the measurement of this strain could play a major role in the design of many new plastic composite applications. Indeed, it may also be possible to not only measure this property but it may also be possible to change the range of this property as desired through compounding and or processing. Such a viscoelastic material development could include modification by forming a new particulate composite or by a combination of compounding and other processing technologies.

It should also be recognized that the constants utilized in this new viscoelastic model can be obtained from creep, constant strain rate, and/or stress relaxation data. Therefore, creep measurements are not necessarily required to determine or confirm this "Projected Elastic Limit."

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